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PHENOMENON OF PREDOMINANT ORIENTATION OF A SUBMERGED ELLIPTICAL CYLINDER UNDER THE ACTION OF SURFACE WAVES

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In many of problems of body rolling caused by waves, it is of interest to study body motion that is established after a long period (compared with the wave period). Body drift is an example of such slow motion. Stationary forces and moments that cause this motion are determined by solving the nonlinear problem of wave–body interaction. To this end, the method of perturbations with the surface-incident-wave amplitude as a small parameter is commonly used. Second-order forces were first studied by Ogilvie [1], who found the vertical drift of a free submerged circular cylinder of neutral buoyancy under the action of surface waves. The second-order theory which is applied in ship hydrodynamics is given in great detail in [2].

An interesting phenomenon observed in studies of horizontal drift of submerged bodies is the possibility of body motion against waves. A solution of the corresponding problem and a literature survey for a circular cylinder are given in [3]. In addition, it is known that waves breaking over an underwater obstacle can also cause its motion against incident waves [4]. For bodies that cross a free surface, horizontal drift motions have been studied most extensively (see, for example, [5]). The question of orientation of three-dimensional elongated bodies is discussed in [6]. It has been shown that in short waves the body is oriented perpendicular to the direction of wave propagation. In long waves another stable orientation parallel to this direction is also possible.

The problem of orientation of a submerged body about the horizontal axis is the least understood. This problem is of great interest, since quite small values of the restoring forces and moments are typical of submerged bodies. The stationary second-order wave moment acting upon a submerged ellipsoid of revolution was determined by Lee and Newman [7]. It has been shown that under the action of incident periodic waves a moment arises that lowers the part of the ellipsoid that first meets the waves.

In this paper, the behavior of a submerged elliptical cylinder which can rotate freely about the horizontal axis under the action of incident waves is studied theoretically and experimentally. Initially, when the liquid is not perturbed, the cylinder is in neutral equilibrium. Special attention is given to the study of the conditions of existence of steady (on the average) states of the cylinder that are consistent with the parameters of the waves incident on the body. The second-order averaged moment was determined theoretically for an infinite reservoir and for the case of reflection from a wave breaker, observed experimentally. It was shown that in the absence of reflected waves the orientation of an elliptical cylinder for a given submersion depends on the wavelength. If reflection occurs, the dependence of the orientation on the reflection coefficient and the phase of the reflected wave becomes more significant. Experimental data are in good agreement with the calculation results. In the experiments we found, in addition, some regimes in which the behavior of the system was dependent on the initial conditions. Under small perturbations, the cylinder oscillates about a certain mean position, while under strong perturbations (shocks) it begins to rotate under the action of waves.

The experimental investigation of the behavior of an elliptical cylinder under the action of waves was carried out in a reservoir which was 4.5 m in length, 0.2 m in width, and 0.8 m in height. A schematic diagram of the setup is shown in Fig. 1. The reservoir had a wave maker 1 of the plunger type, which

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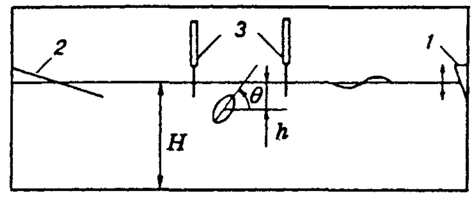

Fig. 1

oscillated sinusoidally, and a wave breaker 2, which was a flat plate tilted at 7°. The lower end of the breaker was submerged 7 cm below the free surface. The water depth in the reservoir H was 24 cm. The elliptical cylinder 19.8 cm in length with 6- and 3-cm axes was placed at a distance of 170 cm from the wave maker with its center submerged $h = 6$ cm below the unperturbed free surface. The cylinder could rotate freely about the geometric center on bearings similar to those used in a watch pendulum. The radial clearance of the bearings did not exceed 0.05 mm. The cylinder was of neutral buoyancy in order to reduce friction. Accurate balancing was performed, so that when the fluid was in the unpertubed state, the cylinder was in neutral equilibrium with respect to the rotation angle θ.

The measurement of the orientation angle of the elliptical cylinder under the action of waves was performed visually by aligning the sighting line of a goniometer placed coaxially with the cylinder outside of the reservoir with the sighting line drawn on the main axis of the elliptical cylinder. The amplitude of steady angular oscillations did not exceed 2°, which allowed the time-averaged orientation angle of the cylinder to be determined with sufficient accuracy. After each measurement the cylinder was turned through 180° to exclude the effect of inaccuracy in balancing. Once the steady state had been reached, the orientation angle was measured again. The spread in values of the angles thus determined did not exceed ±4°. The amplitude and the frequency of incident waves were measured by resistive wavemeters 3 placed to the right and to the left of the cylinder [8]. The wavemeter readings were processed by a computer in real time. Primary attention was given to the cylinder behavior that was established after prolonged operation of the wave maker (from 500 to 2,000 oscillation periods).

Two series of tests were performed. In the first series, the location of the wave maker was not varied. The oscillation frequency ω and amplitude A of the wave maker were varied. The results of the tests are given in Fig. 2. The abscissa is the circular frequency ω of the incident waves and the ordinates are the angles of orientation θ_0 of the main axis of the elliptical cylinder. Points I–III correspond to $A = 1$, 1.5, and 2 cm. The orientation of the cylinder was found to be practically independent of A and hence of the amplitude of the incident surface wave η. The orientation angle of the cylinder changes with the frequency of the incident waves. The frequency ranges of stable orientation are separated by narrow frequency bands (lines 1–5) at which the cylinder begins to rotate, making a full revolution within 6–12 periods of the incident waves. In short waves, the orientation of the main axis of the cylinder is almost vertical. The behavior of the system remains practically unchanged with a change in the submersion level of the cylinder.

The dependence of the cylinder orientation on the wave frequency and the existence of discrete frequencies at which the cylinder begins to rotate can be explained by the presence of waves reflected from the wave breaker.

The behavior of the elliptical cylinder as a function of the reflected wave phase β was studied in the second series of tests. In the case of partial reflection of waves from the wave breaker, the trajectories of the fluid particles are a superposition of trajectories observed for stationary and progressive waves [9]. The reflection coefficient R is defined by the ratio $R = (\eta_1 - \eta_2)/(\eta_1 + \eta_2)$, where η_1 and η_2 are the wave amplitudes at the maximum and minimum regions of the wave envelope, respectively. The measured values of R versus ω are shown in Fig. 3.

The phase of the reflected wave was varied by horizontal displacement of the wave breaker. After decay of the transient process, the orientation angles of the cylinder and the reflection coefficient were measured. The latter remained constant throughout the test. It was found that in long waves with large reflection coefficients

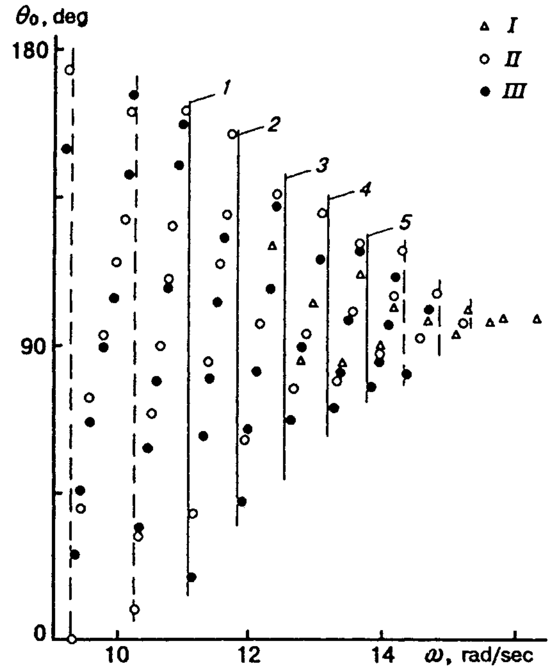

Fig. 2

($R > 0.2$), displacement of the wave breaker leads to changes in orientation of the cylinder from vertical near the minimum of the wave envelope to horizontal near the maximum, and θ_0 changes almost linearly with the phase β of the reflected wave. The orientation of the cylinder has a spatial periodicity, i.e., it changes by 180° when the horizontal coordinate of the wave breaker is changed by a half-wavelength.

For short waves and small reflection coefficients ($R < 0.015$), the orientation angle θ_0 was found to be $100° \pm 4°$ in the absence of wave breaking over the cylinder. In the case of waves of large amplitude, wave breaking occurred and $\theta_0 = 85° \pm 4°$, which is in agreement with the data on the negative drift of underwater obstacles in breaking waves [4].

For intermediate values of R and wavelengths, the behavior of the cylinder was complexly dependent on the wave phase. Near the minimum of the wave envelope (i.e., when the reflected wave phase was $\sim 180°$), the cylinder was stably oriented within a sector of θ_0 close to 90°.

Near the maximum of the wave envelope, no stable orientation of the cylinder exists, and it begins to rotate. Note that rotation of this type with a much longer period than the incident wave period has not been well studied. The literature covers only the rotation of small bodies that have characteristic dimensions smaller than the local amplitude of oscillations of fluid particles. In this case, the rotation frequency is equal to that of the incident waves. A special case of rotation of an airfoil is discussed in [10].

In the intermediate region between the maximum and the minimum of the wave envelope, the behavior of the cylinder depends on the initial conditions. For small perturbations, the cylinder has a stable mean orientation under the action of waves. In the case of strong perturbations (shocks), the cylinder begins to rotate under the action of waves. In this case, the cylinder adjusts the waves so that they maintain this rotation.

Thus, at a given wave frequency, which depends on the horizontal coordinate of the wave breaker, stable orientations of the cylinder exist within a certain sector of angles. For the characteristic R observed in the tests, the experimental relationship between the width of this sector and the frequency of incident waves is shown in Fig. 4. In the absence of strong perturbations (shocks), the range of stable values of θ_0 is bounded by curves 1. The range of values of θ_0 that are stable against strong pertubations is narrower and is shown by curves 2. The dots show the experimental data. For the angles between curves 1 and 2, "severe" loss of

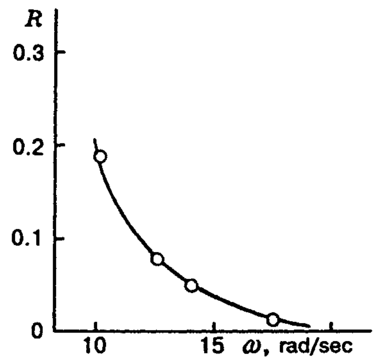

Fig. 3

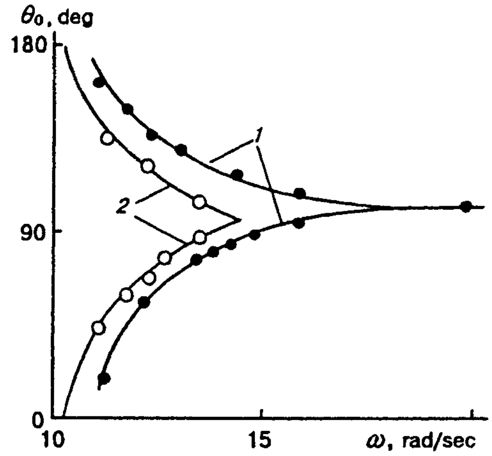

Fig. 4

stability is typical. The higher the wave frequency, the narrower the sector of the angles realized. As follows from Fig. 4, the orientation angle θ_0 becomes independent of the reflected wave phase for $\omega > 18$ rad/sec and $R < 0.015$.

From a physical point of view, two factors govern the behavior of the cylinder: 1) the influence of the stationary component of the wave and 2) the suction force due to high-speed flow around the edge of the elliptical cylinder. The second factor determines the orientation of the cylinder in short waves. Near the minimum of the wave envelope, both effects act in the same direction and lead to stable vertical orientation of the cylinder. Near the maximum of the wave envelope the effects act in opposite directions, and this leads to rotation of the cylinder.

The theoretical solution is based on the hypothesis that the value of the mean angle θ_0 for steady angular oscillations of the cylinder corresponds to the value at which the second-order moment $\overline{M^{(2)}}$ averaged over the oscillation period is equal to zero and its derivative $\partial\overline{M^{(2)}}/\partial\theta|_{\theta=\theta_0}$ is negative (positive) if the rules of signs for the moment and for the angle are the same (different). Using the linear theory of rolling with regular waves, we determine the average second-order moment acting on a submerged elliptical cylinder.

Let us consider the two-dimensional problem for a semi-infinite homogeneous fluid. It is assumed that the fluid is nonviscous and incompressible and that its flow is potential. The velocity potential $\Phi_0(x, y, t)$ for a surface wave incident from the right is

$$\Phi_0 = \eta \mathrm{Re}\,[\psi_0 \exp(i\omega t)], \tag{1}$$

where $\psi_0(x, y) = ig\exp[k(y + ix)]/\omega$; $k = \omega^2/g$ is the wave number; and g is the acceleration of gravity. The x axis of the fixed coordinate system coincides with the unperturbed free surface and the y axis is directed vertically upward.

Under the action of this wave a submerged elliptical contour with fixed center begins to move. It is assumed that with time it enters the regime of angular oscillations about a certain mean position with the main axis inclined at angle θ to the horizon. We write the equation for the elliptical surface at its mean position as $S_0(x, y) = 0$, where

$$S_0 = (x\cos\theta + y_1\sin\theta)^2/a^2 + (x\sin\theta - y_1\cos\theta)^2/b^2 - 1;$$

a and b are the major and minor semiaxes of the ellipsoid, respectively; $y_1 = y + h$; and h is the depth of submersion of its center.

The velocity potential $\Phi(x, y, t)$ of the fluid motion satisfies the Laplace equation

$$\Delta\Phi = 0 \tag{2}$$

with the boundary conditions on the free surface $y = Y(x,t)$

$$\frac{\partial\Phi}{\partial x}\frac{\partial Y}{\partial x} - \frac{\partial\Phi}{\partial y} + \frac{\partial Y}{\partial t} = 0; \tag{3}$$

$$\frac{\partial\Phi}{\partial t} + \frac{1}{2}|\nabla\Phi|^2 + gY = 0, \tag{4}$$

and on the body surface $S(x,y,t) = 0$

$$\nabla\Phi\nabla S + \frac{\partial S}{\partial t} = 0. \tag{5}$$

The function S, which describes the instantaneous position of the body, is

$$S = (xc_1 + y_1 s_1)^2/a^2 + (xs_1 - y_1 c_1)^2/b^2 - 1.$$

Here $c_1 = \cos(\theta + \vartheta(t))$; $s_1 = \sin(\theta + \vartheta(t))$; the function $\vartheta(t)$ describes small angular deviations of the main axis of the ellipsoid from its mean position. The amplitude of the incident surface wave is assumed to be small, and, according to the method of perturbations, each dependent variable can be represented as a series in a certain small parameter ε. Thus, the series for the function $\Phi(x,y,t)$ is written as

$$\Phi(x,y,t) = \varepsilon\Phi^{(1)}(x,y,t) + \varepsilon^2\Phi^{(2)}(x,y,t) + \dots . \tag{6}$$

We also assume that the boundary conditions at two nonstationary boundaries can be extended to unperturbed positions of these boundaries by means of Fourier series. Substituting series (6) into Eq. (2) and boundary conditions (3)–(5) and using the representation for the potential adopted in the theory of rolling,

$$\Phi^{(1)}(x,y,t) = \mathrm{Re}\left\{[\eta(\psi_0 + \psi_D) + \zeta\psi_R]\exp(i\omega t)\right\},$$

we have the following problem for first-order terms:

$$\Delta\psi_D = 0, \quad \Delta\psi_R = 0, \quad \omega^2\psi_D = g\partial\psi_D/\partial y, \quad \omega^2\psi_R = g\partial\psi_R/\partial y \quad (y = 0),$$
$$\partial\psi_D/\partial n = -\partial\psi_0/\partial n, \qquad \partial\psi_R/\partial n = -i\omega\mathbf{r}\times\mathbf{n} \qquad (x, y \in S_0).$$

Here ψ_D is the diffraction potential, which describes the wave motion of the fluid resulting from scattering of an incident wave with potential ψ_0 by a fixed elliptical contour with orientation angle θ; ψ_R is the radiation potential due to rotational oscillations of the body that obey the law $\vartheta^{(1)}(t) = \mathrm{Re}[\zeta\exp(i\omega t)]$; ζ is the complex amplitude; $\mathbf{r} = (x, y_1)$ is the radius vector of a point on the ellipsoid surface with respect to the fixed center of the ellipsoid; $\mathbf{n}$ is the inward normal to the body surface. In the far field, the conditions of radiation decay of the wave process for $y \to -\infty$ should be satisfied.

In the problem considered the motion of the body is due only to the moment of hydrodynamic forces that acts on the body: $M(t) = \int_{S=0} p\mathbf{r}\times\mathbf{n}\,ds$. The pressure $p(x,y,t)$ is determined ignoring the hydrostatic forces:

$$p = -\rho\left(\frac{\partial\Phi}{\partial t} + \frac{1}{2}|\nabla\Phi|^2\right) \tag{7}$$

(ρ is the fluid density). A positive value of the moment causes counterclockwise motion of the body. The functions p and M can also be expanded into a series in ε similar to (6):

$$p = \varepsilon p^{(1)} + \varepsilon^2 p^{(2)} + \dots, \qquad M = \varepsilon M^{(1)} + \varepsilon^2 M^{(2)} + \dots .$$

Using (7) we obtain:

$$p^{(1)} = -\rho\frac{\partial\Phi^{(1)}}{\partial t}, \qquad p^{(2)} = -\rho\Big(\frac{\partial\Phi^{(2)}}{\partial t} + \frac{1}{2}|\nabla\Phi^{(1)}|^2\Big).$$

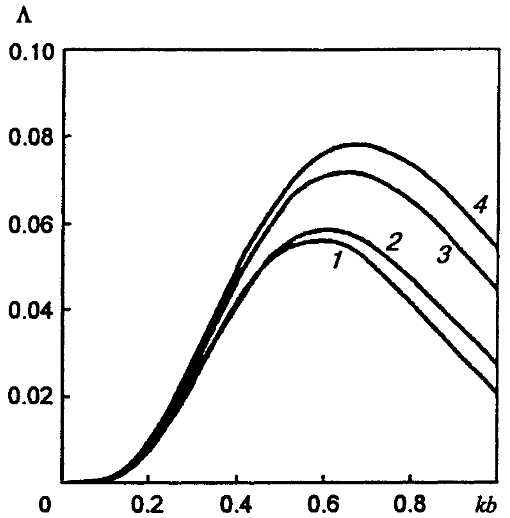

Fig. 5

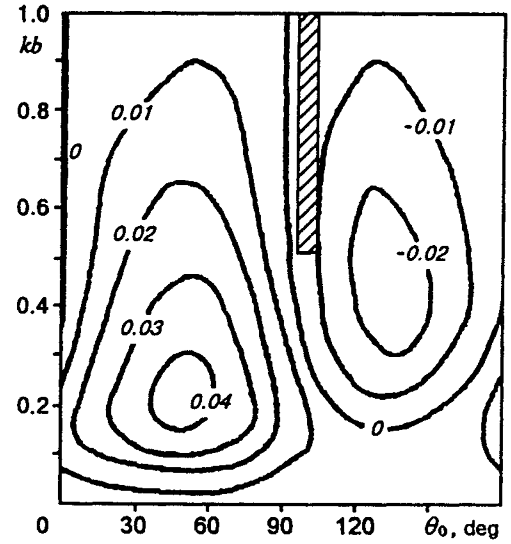

Fig. 6

Replacing the integration over the instantaneous position of the body $S = 0$ by integration over the contour $S_0 = 0$ in calculations of the moment (details see, for example, [1]), we obtain

$$M^{(1)} = -\rho \int\limits_{S_0=0} \frac{\partial \Phi^{(1)}}{\partial t} \mathbf{r} \times \mathbf{n}\, ds,$$

$$M^{(2)} = -\rho \int\limits_{S_0=0} \left[\frac{\partial \Phi^{(2)}}{\partial t} + \frac{1}{2}|\nabla \Phi^{(1)}|^2 - \vartheta^{(1)}\Big(x\frac{\partial^2 \Phi^{(1)}}{\partial y \partial t} - y_1 \frac{\partial^2 \Phi^{(1)}}{\partial x \partial t}\Big)\right] \mathbf{r} \times \mathbf{n} ds.$$

It is known [2] that because of the sinusoidal oscillations of the function $\Phi^{(1)}$ the value of the second-order moment $\overline{M^{(2)}}$ averaged over the oscillation period does not depend on the second-order potential and is determined only by first-order solutions.

The linear theory of rolling of a submerged cylinder has been well studied, and there are a number of numerical methods for solution of this problem (see, for instance, [11, 12] and the references therein). The moment of hydrodynamic forces in a linear approximation is $M^{(1)}(t) = \mathrm{Re}[(F_D + F_R)\exp(i\omega t)]$, where the diffraction moment is

$$F_D = -i\omega\rho\eta \int\limits_{S_0=0} (\psi_0 + \psi_D)\mathbf{r} \times \mathbf{n}\ ds,$$

and the radiation moment is

$$F_R = -i\omega\rho\zeta \int\limits_{S_0=0} \psi_R\, \mathbf{r} \times \mathbf{n}\, ds = \zeta\,(\omega^2 \mu - i\omega\lambda).$$

The real μ and λ are known as the added mass and damping coefficients, respectively.

Using the equation of rotation of a solid body about a fixed axis $I d\vartheta^{(1)}/dt = M^{(1)}$ (I is the moment of inertia of the ellipse about the rotation axis) we obtain: $\zeta = F_D/[i\omega\lambda - \omega^2(I + \mu)]$. This leads to the final expression for $\overline{M^{(2)}}$

$$\overline{M^{(2)}} = -\frac{\rho}{2} \int\limits_{S_0=0} \left\{\frac{1}{2}|\eta\frac{\partial \psi_D}{\partial s} + \zeta\frac{\partial \psi_R}{\partial s}|^2 + \omega \mathbf{r}\cdot \mathbf{n} \mathrm{Re}\Big[i\zeta^* \Big(\eta\frac{\partial \psi_D}{\partial s} + \zeta\frac{\partial \psi_R}{\partial s}\Big)\Big]\right\} \mathbf{r} \times \mathbf{n}\, ds$$

(here the asterisk denotes the complex conjugate).

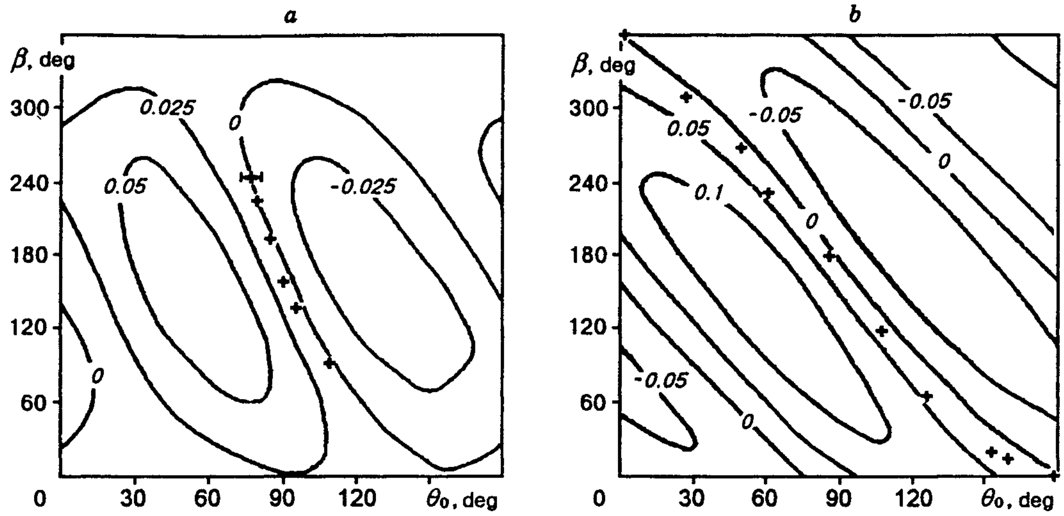

Fig. 7

The first-order radiation and diffraction problems were solved numerically using the coupled finite-element method for $a/b = 2$ and $h/b = 4$ for various angles of attack and various wavelengths. This method is described in detail in [11, 12]. The number of finite elements in the calculations was 18. The dependence of the first-order characteristics μ, λ, and F_D on the slope of the ellipse is of special interest. The damping coefficient undergoes the greatest changes.

Values of $\Lambda = \lambda\omega/\rho g b^3$ for angles $\theta = 0$, 30, 60, and 90° are given in Fig. 5 (curves 1–4). Values are given only for $\theta \leqslant \pi/2$, since it can be easily shown that $\lambda(k,\theta) = \lambda(k, \pi - \theta)$. The same relationship is also valid for the added mass coefficient μ. The latter, however, depends only slightly on θ, and its relative deviation from the case of $\theta = 0$ does not exceed 3% in the indicated range of parameter variation. The diffraction moment is known to be related to the damping coefficient by the Haskind–Newman relation (for details see, for example, [12]).

Curves of equal value for the function $M_2 = \overline{M^{(2)}}/\rho\eta^2 g b$ are shown in Fig. 6. Because of nonuniformity of the elliptical cylinder in the experiment, the moment of inertia of the ellipse in the calculations was specified as $I = 1.16 I_0$, where $I_0 = \pi \rho_b a b (a^2 + b^2)/4$. The average density ρ_b of the cylinder material was practically (with accuracy to 1%) equal to the water density ρ. It is evident that for long waves the average second-order moment is positive, and the elliptical cylinder rotates in the counter-clockwise direction under the action of waves with wave numbers $kb \leqslant 0.15$. Under the action of short waves ($kb > 0.6$) in the stationary regime the cylinder oscillates about its vertical ($\theta_0 = 90°$) position. In the intermediate range of wavelengths, the cylinder (on the average) takes an inclined position with orientation angle $90° \leqslant \theta_0 \leqslant 130°$, which depends on the wavelength. In Fig. 6, the hatched region shows experimental data according to which the cylinder is oriented at an angle $\theta_0 = 100° \pm 4°$ for $kb > 0.5$.

In the theoretical solution one can also take into account the combined action of an incident wave with potential (1) and a reflected wave with velocity potential $\widetilde{\Phi}_0 = \eta \mathrm{Re}[\widetilde{\psi}_0 \exp(i\omega t)]$, $\widetilde{\psi}_0(x,y) = ig\alpha \exp[k(y - ix)]/\omega$, where the complex value $\alpha = R\exp(i\beta)$ determines the reflection coefficient R and the phase of this wave β. Proceeding from the type of reflected wave, one can easily show that the diffraction potential of the reflected wave $\widetilde{\Phi}_D$ is related to the analogous parameter of the incident wave by the simple relation $\widetilde{\psi}_D = -\psi_D^*$. The first-order moment in this case is

$$M^{(1)}(t) = -\rho\omega \mathrm{Re}\{i \exp(i\omega t) \int\limits_{S_0=0} [\eta(\psi_0 + \psi_D + \alpha(\psi_0^* + \psi_D^*)) + \zeta\psi_R]\, \mathbf{r} \times \mathbf{n}\, ds\},$$

the amplitude of forced oscillations of the body in the presence of a reflected wave is

$$\zeta = (F_D + \alpha F_D^*)/[i\omega\lambda - \omega^2(I + \mu)],$$

and the mean second-order moment is

$$\overline{M^{(2)}} = -\frac{\rho}{2} \int\limits_{S_0=0} \left\{ \frac{1}{2} \left[\eta^2 (1 + R^2) |\frac{\partial \psi_D}{\partial s}|^2 + |\zeta \frac{\partial \psi_R}{\partial s}|^2 \right] \right.$$

$$\left. + \mathrm{Re}\Big[\eta \left(\zeta \frac{\partial \psi_R}{\partial s} \left(\frac{\partial \psi_D^*}{\partial s} - \alpha^* \frac{\partial \psi_D}{\partial s} \right) - \alpha^* \eta \left(\frac{\partial \psi_D}{\partial s} \right)^2 \right) + i\omega\zeta^* \mathbf{r} \cdot \mathbf{n} \Big(\eta \left(\frac{\partial \psi_D}{\partial s} - \alpha \frac{\partial \psi_D^*}{\partial s} \right) + \zeta \frac{\partial \psi_R}{\partial s} \Big) \Big] \right\} \mathbf{r} \times \mathbf{n} \, ds.$$

The following values of wave numbers and reflection coefficients were selected from the experimental data: 1) $kb = 0.47$, $R = 0.014$, 2) $kb = 0.24$, $R = 0.08$, 3) $kb = 0.16$, $R = 0.19$. The dependence of M_2 on the slope of the ellipse and on the phase of the reflected wave was calculated. In the first case there is no influence of the phase β, because of the small contribution of the reflected wave. The results for the second and third cases one are given in Figs. 7a and 7b, respectively (crosses show the experimental data and their dispersion). An interesting feature of the third case is that for any phase of the reflected wave, the cylinder oscillates about a certain mean angle θ_0 which depends almost linearly on β.

This comparison shows that the hypothesis proposed here is supported by experimental data.

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